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## Midterm exam Introduction to Logic (CS)

Thursday 5 October 2017, 9 - 11 h.

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- Write your student number and your tutorial group on the first page. **Do not write your name** so as to enable anonymous grading.
  - Write with a *blue or black pen* (so no pencil, no red pen).
  - With the regular exercises, you can earn 100 points. With the bonus exercise, you can earn additionally 10 points.  
The grade is the number of points you earned divided by 10, with a maximum of 10.
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1. (10 points)

Translate the following sentences to *propositional logic*, where atomic sentences are represented by upper-case letters. Do not forget to provide the translation key.

- (a) I like your soup, unless it is green and does not contain noodles.
- (b) We will reach our destination only if we work together and have some luck.

2. (10 points)

Translate the following sentences to *first-order logic*. Do not forget to provide the translation key.

- (a) Evi and Alex will help Suzanne, provided that Mircea will help Evi and Maaïke.
- (b) Neither Maaïke nor Alex will be in time if Evi sends Mircea to Suzanne.

3. (30 points)

Give formal proofs for the following inferences. Do not forget to provide justifications. You may only use the Introduction and Elimination rules and the Reiteration rule.

- (a) 
$$\left| \begin{array}{l} \neg P \rightarrow P \\ \hline P \end{array} \right.$$
- (b) 
$$\left| \begin{array}{l} P \rightarrow (Q \rightarrow (R \rightarrow S)) \\ \hline (P \wedge Q \wedge R) \rightarrow S \end{array} \right.$$
- (c) 
$$\left| \begin{array}{l} (P(b) \vee a = b) \rightarrow (P(a) \wedge a = b) \\ \hline P(b) \leftrightarrow a = b \end{array} \right.$$

4. (20 points)

Use *truth tables* to answer the next questions. Make the full truth table, and motivate your answer. Order the rows in the truth tables as follows:

$P$	$Q$	$R$	...
T	T	T	...
T	T	F	...
T	F	T	...
T	F	F	...
F	T	T	...
F	T	F	...
F	F	T	...
F	F	F	...

$a = a$	$A(a)$	$A(b)$	...
T	T	T	...
T	T	F	...
T	F	T	...
T	F	F	...
F	T	T	...
F	T	F	...
F	F	T	...
F	F	F	...

- (a) Is  $(P \leftrightarrow \neg Q) \vee \neg(Q \wedge R)$  a tautology?
- (b) Are the following sentences *logically equivalent*? Indicate the spurious rows in the truth table.
- i.  $A(a) \wedge \neg(A(b) \wedge a = a)$
  - ii.  $\neg(A(a) \rightarrow (a = a \rightarrow A(b)))$

5. (20 points)

- (a) Provide a negation normal form (NNF) of the sentence  $\neg(P \vee \neg(\neg Q \rightarrow R))$ .
- (b) Provide a conjunction normal form (CNF) of the sentence  $\neg(P \rightarrow \neg(Q \rightarrow R))$ .

Indicate in both cases the intermediate steps. You do not have to provide justifications for the steps.

6. (10 points)

Given the following three sets  $A = \{0\}$ ,  $B = \{0, 1, 2, 3\}$  and  $C = \{2, 3, 4, 5\}$ . For each of the following statements, determine whether it is true or false. You are not required to explain or justify the answer.

- |   |                                 |
|---|---------------------------------|
| (a) $(B \setminus C) \setminus A = \{1\}$ | (f) $4 \in (A \cup B) \cap C$   |
| (b) $2 \notin (A \cap B) \cup (B \cap C)$ | (g) $A \cap B = \emptyset$      |
| (c) $\emptyset \in A$                     | (h) $\emptyset \not\subseteq A$ |
| (d) $A \setminus B \subseteq C$           | (i) $A \subseteq B \setminus C$ |
| (e) $(B \cap C) \cup A \subseteq B$       | (j) $A \in B$                   |

7. (Bonus exercise: 10 points)

Give a formal proof for  $(P \rightarrow Q) \vee (Q \rightarrow P)$ . Do not forget to provide justifications. You may only use the Introduction and Elimination rules and the Reiteration rule.